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Some New Generators to Produce Minimal Circular Strongly Balanced Generalized Neighbor Designs of Class-I

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Abstract Minimal circular strongly balanced neighbor designs control the neighbor effects economically but these designs can only be constructed for v odd. For v even, their alternates are minimal circular strongly balanced generalized neighbor designs of class-I (MCSBGNDs-I) which are the designs in which (i) each treatment appears exactly once with itself as neighbors, (ii) $\frac{v}{2}$ unordered pairs appear twice as neighbors while the remaining ones appear once. On the basis of constructors developed by Munir et al. (2023) [15], some new generators are developed in this article to generate cyclic shifts for efficient MCSBGNDs-I in blocks of (i) equal sizes, (ii) two different sizes, and (iii) three different sizes for m (mod 4) $\equiv$ 2 & 3 with v even and $k = 4l$, $k = 4l+2$, k (odd) > 3, k (mod) $\equiv$ 1 & k (mod) $\equiv$ 3. Efficiency of neighbor effects and of Separability show that our proposed generators produce designs which control the neighbor effects efficiently as well as estimate the direct effects and neighbor effects independently.

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1 Introduction

In most of the variety competition experiments, neighbor effects become the source of bias. Minimal circular strongly balanced neighbor designs (MCSBNDs) minimize this bias economically but these designs can only be constructed for v odd. For v even, minimal circular strongly balanced generalized neighbor designs of class-I (MCSBGNDs-I) are considered as alternates to MCSBNDs. MCSBGNDs-I are the designs in which $\frac{v}{2}$ unordered pairs $(0, \frac{v}{2}), (1, \frac{v+2}{2}), \cdots, (\frac{v-2}{2}, v-1)$ appear twice as neighbors while all others appear once including $(0,0), (1,1), \cdots, (v-1, v-1)$. Rees (1967) [23] presented neighbor designs (NDs) for v odd. Hwang (1973) [9] presented some infinite classes of neighbor designs for v odd. Mettei (1996) [14] presented balanced NDs for $v = 2m$ and $k = m$. Ahmed and Akhtar (2008) [1] also developed some series of NDs. Ahmed and Akhtar (2011) [2] presented these design for $k = 6$.

Hamad (2014) [6], Hamad and Hanif (2016) [7] constructed some partially balanced NDs. Nadeem et al. (2021) [19] constructed economical classes of MCGNDs. Salam et al. (2021) [25] presented some new constructions of MCBNDs then Salam et al. (2022) [24] presented MCNSBNDs. Sharif et al. (2022) [27] constructed quasi Rees neighbor designs. Noreen et al. (2022) [20] developed some generators which generate cyclic shifts to obtain MCSPBNDs-II for all v even. Mehmood et al. (2022) [13] developed some generators to obtain MCWBNDs-II for all v even with $v = 2ik_1 + 2k_2 - 2$. Rasheed et al. (2022) [22] developed some generators which generates cyclic shifts to obtain MCWBNDs-II for $v = 2ik_1 + 2k_2 - 2$ with m (mod 4) $\equiv$ 1 & 2. Nadeem et al. (2022a) [16] developed some generators to obtain MCPBNDs-I with m (mod) $\equiv$ 0 & 3. Nadeem et al. (2022b) [18] developed constructors to obtain MCPBNDs-II with m (mod 4) $\equiv$ 0 & 1 for $k_1 = 4l$, $k_1 = 4l + 2$, k_1 (odd) > 3, k_1 (mod) $\equiv$ 1, k_1 (mod) $\equiv$ 3. Nadeem et al. (2023) [17] developed some generators to obtain MCPBNDs-I for $v = 2ik_1 + 4k_2 + 2$ with $k_2 = 3, 4$ & 5. Ali et al. (2023) [3] developed some generators which generate cyclic shifts to obtain MCSPBNDs-I with m (mod 4) $\equiv$ 1 & 2. Mehmood et al. (2023) [12] developed constructors to obtain MCPBNDs-II with m (mod 4) $\equiv$ 2 & 3. Shabbir et al. (2023) [26] developed R-coded algorithm for CGN_2-designs. Hassan et al. (2023) [8] constructed efficient MCWBNDs in blocks of three different sizes. Fardos et al. (2024) [5] developed an algorithm to construct MCBNDs and its efficient classes. Munir et al. (2023) [15] developed the following constructors for MCSBGNDs-I.

- **A** $= [0, 1, 2, \cdots, m, m+1]$ for m (mod 4) $\equiv 2$, where $m = \frac{v-2}{2}$ & v even.
- **B** $= [0, 1, \cdots, \frac{m-3}{4}, \frac{m+5}{4}, \frac{m+9}{4}, \cdots, m, m+1, \frac{7(m+1)}{4}]$ for m (mod 4) $\equiv 3$.

On the basis of the above mentioned constructors, some new generators are developed in this article to generate cyclic shifts for MCSBGNDs-I in blocks of (a) Equal sizes for $v = 2ik - 2$, (b) Two different sizes for (i) $v = 2ik_1 + 2k_2 - 2$, (ii) $v = 2ik_1 + 4k_2 - 2$, and (c) Three different sizes for (i) $v = 2ik_1 + 2k_2 + 2k_3 - 2$, (ii) $v = 2ik_1 + 4k_2 + 2k_3 - 2$, (ii) $v = 2ik_1 + 2k_2 + 4k_3 - 2$, (iii) $v = 2ik_1 + 4k_2 + 4k_3 - 2$.

2 Method of Construction

In this article, generators to obtain the MCSBGNDs-I are developed under the logic of method of cyclic shifts (Rule I) introduced by Iqbal (1991) [10] which is explained as:

Let $S_j = [q_{j1}, q_{j2}, \cdots, q_{j(k-1)}]$; be l sets, where $j = 1, 2, \cdots, l$ with $0 \leq q_{ji} \leq v-1$. If each of $0, 1, \cdots, v-1$ appears once in S^* but $\frac{v}{2}$ appears twice with v even then these sets will produce MCSBGND-I, where $S^* = [q_{j1}, q_{j2}, \cdots, q_{j(k-1)}, (q_{j1} + q_{j2} + \cdots + q_{j(k-1)}) \bmod v, v - q_{j1}, v - q_{j2}, \cdots, v - q_{j(k-1)}, v - (q_{j1} + q_{j2} + \cdots + q_{j(k-1)}) \bmod v]$.

Example 1. *$S_1 = [1, 6, 7]$ and $S_2 = [3, 4, 5]$ produce MCSBGND-I for $v = 14$ and $k = 4$.*

Proof. $S^* = [1, 6, 7, \mathbf{0}, 3, 4, 5, \mathbf{12}, 13, 8, 7, 11, 10, 9, 2]$, Here each of $0, 1, 2, \cdots, 13$ appears once except 7 which appear twice. Hence $S_1 = [1, 6, 7]$ and $S_2 = [3, 4, 5]$ produce MCSBGND-I for $v = 14$, $k = 4$.

To generate the design, use v blocks for S_1. Write $0, 1, \cdots, v-1$ in Row 1. Add 1st value of S_1 (mod v) to Row 1 to get Row 2. Similarly, add 2nd value of S_1 (mod v) to Row 2 to get Row 3, and so on, see Table 1.

Table 1. Blocks obtained from $S_1 = [1, 6, 7]$

Blocks													
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
0	1	2	3	4	5	6	7	8	9	10	11	12	13
1	2	3	4	5	6	7	8	9	10	11	12	13	0
7	8	9	10	11	12	13	0	1	2	3	4	5	6
0	1	2	3	4	5	6	7	8	9	10	11	12	13

Use v more blocks for S_2 and obtain the design, see Table 2.

Table 2. Blocks obtained from $S_2 = [3, 4, 5]$

Blocks													
15	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**
0	1	2	3	4	5	6	7	8	9	10	11	12	13
3	4	5	6	7	8	9	10	11	12	13	0	1	2
7	8	9	10	11	12	13	0	1	2	3	4	5	6
12	13	0	1	2	3	4	5	6	7	8	9	10	11

Table 1 and Table 2 jointly produce the complete design generated through $S_1 = [1, 6, 7]$ and $S_2 = [3, 4, 5]$ for $v = 14$ & $k = 4$, using 28 blocks. □

Example 2. $S_1 = [3, 4, 5, 8, 10]$, $S_2 = [7, 11, 12, 13, 15]$, $S_3 = [6, 9, 14]$ *produce MCSBGNDs-I for* $v = 30$, $k_1 = 6$ *and* $k_2 = 4$.

Proof. $S^* = [3, 4, 5, 8, 10, \mathbf{0}, 7, 11, 12, 13, 15, \mathbf{28}, 6, 9, 14, \mathbf{29}, 27, 26, 25, 22, 20, 23, 19, 18, 17, 15, 2, 24, 21, 16]$, Here each of $0, 1, \cdots, 29$ appears once except 15 which appear twice. Hence $S_1 = [3, 4, 5, 8, 10]$, $S_2 = [7, 11, 12, 13, 15]$, $S_3 = [6, 9, 14]$ produce MCSBGNDs-I for $v = 30$, $k_1 = 6$ and $k_2 = 4$. □

Example 3. $S_1 = [3, 4, 6, 8]$, $S_2 = [2, 9, 11]$, $S_3 = [7, 10]$ *produce MCSBGNDs-I for* $v = 22$, $k_1 = 5$, $k_2 = 4$ *and* $k_3 = 3$.

Proof. $S^* = [3, 4, 6, 8, \mathbf{21}, 2, 9, 11, \mathbf{0}, 7, 10, \mathbf{17}, 19, 18, 16, 14, 1, 20, 13, 11, 15, 12, 5]$, Here each of $0, 1, 2, \cdots, 21$ appears once except 11 which appear twice. Hence $S_1 = [3, 4, 6, 8]$, $S_2 = [2, 9, 11]$, $S_3 = [7, 10]$ produce MCSBGNDs-I for $v = 22$, $k_1 = 5$, $k_2 = 4$ and $k_3 = 3$. □

3 Efficiency Measures

3.1 Efficiency of neighbor Effects (residual effects)

Canonical efficiency factors are Eigen values (non-zero) of information matrix C^*, and efficiency factor for both direct and residual effect is harmonic mean of Eigen values (non-zero) of their respective information matrix, see James and Wilkinson (1971) [11] and Pearce et al. (1974) [21]. If the canonical factors are $\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_v$ then following is the formula to find E_n. Design will be efficient to estimate neighbor effects if value of E_n is high.

$$E_n = \left[\frac{1}{v-1}\sum_{i=1}^{v-1}\frac{1}{\varepsilon_i}\right]^{-1}$$

3.2 Efficiency of Separability

Divecha and Gondaliya (2014) [4] developed following efficiency of Separability (E_S) for balanced and weakly balanced neighbor designs.

$$E_S = \left[1 - \frac{1}{v\sqrt{v-1}}\right] \times 100\%$$

4 Generators for MCSBGNDs-I in Equal Block Sizes for v = 2ik – 2

In this Section, generators are developed to obtain MCSBGNDs-I in equal block sizes.

4.1 MCSBGNDs-I can be constructed for v = 2ik – 2 when m (mod 4) ≡ 2 with

- $k = 4l$, i integer.
- $k = 4l + 2$, i even.
- k (odd) > 3, i (mod 4) ≡ 0.

Example 4. *MCSBGNDs-I for $v = 2ik - 2$ with m (mod 4) ≡ 2. See Table 3.*

Table 3. MCSBGNDs-I for $v = 2ik - 2$ with m (mod 4) ≡ 2.

v	k	Sets of Shifts	E_n	E_s
14	4	[1,6,7]+[3,4,5]	0.57	0.77
22	6	[5,8,9,10,11]+[2,3,4,6,7]	0.61	0.81
38	5	[4,10,11,13]+[3,5,14,15]+[7,8,9,12]+[16,17,18,19]	0.72	0.85

4.2 MCSBGNDs-I can be constructed for v = 2ik – 2 when m (mod 4) ≡ 2 with

- k (mod 4) ≡ 1, i (mod 4) ≡ 1.
- k (mod 4) ≡ 3, i (mod 4) ≡ 3.

Example 5. *MCSBGNDs-I for $v = 2ik - 2$ with m (mod 4) ≡ 2. See Table 4.*

Table 4. MCSBGNDs-I for $v = 2ik - 2$ with m (mod 4) ≡ 2.

v	k	Sets of Shifts	E_n	E_s
48	5	[2,3,20,23]+[13,16,21,42]+[7,10,12,18]+[8,9,11,15]+[17,19,22,24]	0.74	0.87
40	7	[8,9,10,14,15,17]+[11,13,18,19,20,35]+[1,2,3,6,12,16]	0.79	0.86

5 Generators for MCSBGNDs-I in Block of Two Different Sizes

In this Section, generators are developed to obtain MCSBGNDs-I in two different block sizes.

5.1 MCSBGNDs-I can be constructed in Block of Two Different Sizes for $v = 2ik_1 + 2k_2 - 2$ when m (mod 4) ≡ 2 with

- $k_1 = 4l$, $k_2 = 4$, i integer, $l > 1$.
- $k_1 = 4l + 2$, (i) $k_2 = k_1 - 2$, i even, (ii) $k_2 = 4$, i even.
- k_1 (odd) > 3, (i) i (mod 4) ≡ 1, $k_2 = k_1 - 2$, (ii) $k_2 = 4$, i (mod 4) ≡ 0.
- k_1 (mod 4) ≡ 1, (i) i (mod 4) ≡ 0, $k_2 = k_1 - 1$, (ii) i (mod 4) ≡ 1, $k_2 = 3$, (iii) i (mod 4) ≡ 3, $k_2 = 5$.
- k_1 (mod 4) ≡ 3, (i) i (mod 4) ≡ 2, $k_2 = k_1 - 1$, (ii) i (mod 4) ≡ 3, $k_2 = 3$, (iii) i (mod 4) ≡ 1, $k_2 = 5$.

Example 6. *MCSBGNDs-I for $v = 2ik_1 + 2k_2 - 2$ with m (mod 4) ≡ 2. See Table 5*

Table 5. MCSBGNDs-I for $v = 2ik_1 + 2k_2 - 2$ with m (mod 4) ≡ 2.

v	k_1	k_2	Sets of Shifts	E_n	E_s
54	6	4	[4,5,13,22]+[7,8,15,16]+[6,10,12,17]+[19,20,21,23]+[11,14,18]	0.73	0.87
38	7	6	[4,6,10,16,18,19]+[5,11,12,14,15,17]+[1,7,8,9,13]	0.78	0.85
30	6	4	[3,4,5,8,10]+[7,11,12,13,15]+[6,9,14]	0.64	0.84
14	5	3	[1,3,4,6]+[5,7]	0.60	0.78
22	9	3	[1,3,4,5,6,7,8,10]+[9,11]	0.62	0.81
46	7	3	[4,10,12,20,21,22]+[6,11,14,15,18,23]+[1,2,7,8,9,19]+[16,18]	0.65	0.84
22	8	4	[1,2,3,5,6,8,9,11]+[4,7,10]	0.85	0.76
46	10	4	[5,6,11,23]+[7,8,15,16]+[19,20,21,22]+[4,9,14,17]+[12,13,18]	0.75	0.87
62	7	4	[5,6,11,23]+[7,8,15,16]+[19,20,21,22]+[4,9,14,17]+[12,13,18]	0.73	0.87
62	9	5	[3,4,9,15,19,21,22,31]+[6,8,10,12,17,20,23,26]+ [11,13,16,25,27,28,29,30]+[5,14,18,24]	0.78	0.88
30	11	5	[3,4,6,7,9,10,11,12,13,15]+[2,5,8,14]	0.69	0.79

5.2 MCSBGNDs-I can be constructed in Two Different Block Sizes for $v = 2ik_1 + 2k_2 - 2$ when m (mod 4) ≡ 3 with

- $k_1 = 4l + 2$, (i) $k_2 = k_1 - 1$, i even, (ii) $k_2 = 3$, i odd, (iii) $k_2 = 5$, i even.
- k_1 (odd) > 3, (i) i (mod 4) ≡ 1, $k_2 = k_1 - 1$, (ii) $k_2 = 3$, i (mod 4) ≡ 2, (iii) $k_2 = 5$, i (mod 4) ≡ 0.
- k_1 (mod 4) ≡ 1, (i) i (mod 4) ≡ 2, $k_2 = k_1 - 2$, (ii) k_1 (mod 4) ≡ 3, i (mod 4) ≡ 0, $k_2 = k_1 - 2$.
- k_1 (mod 4) ≡ 1, (i) i (mod 4) ≡ 1, $k_2 = 4$, (ii) i (mod 4) ≡ 3, $k_2 = 4$.

Example 7. *MCSBGNDs-I for $v = 2ik_1 + 2k_2 - 2$ with m (mod 4) ≡ 3. See Table 6.*

Table 6. MCSBGNDs-I for $v = 2ik_1 + 2k_2 - 2$ with m (mod 4) ≡ 3.

v	k_1	k_2	Sets of Shifts	E_n	E_s
32	6	5	[8,10,12,13,14]+[3,5,11,16,28]+[2,6,915]	0.74	0.82
16	5	4	[4,5,8,14]+[3,6,7]	0.63	0.79
24	5	3	[1,6,7,10]+[5,9,11,21]+[8,12]	0.67	0.82
64	7	5	[3,11,17,19,22,56]+[9,16,20,25,28,29]+[7,13,23,24,26,30]+ [12,15,18,21,27,31]+[6,10,14,32]	0.68	0.93
16	6	3	[1,4,6,7,14]+[5,8]	0.64	0.79
32	7	3	[7,8,10,11,12,13]+[2,5,6,9,14,28]+[15,16]	0.75	0.84
24	9	4	[1,5,6,7,9,11,12,21]+[4,8,10]	0.75	0.82
48	7	4	[4,14,16,17,22,23]+[3,8,9,13,19,42]+[7,11,15,18,20,24]+[10,12,21]	0.78	0.87
48	10	5	[2,3,8,9,10,11,14,18,20]+[4,7,12,15,16,21,22,23,24]+[13,17,19,42]	0.83	0.87
80	9	5	[12,23,27,29,32,38,39,40]+[5,9,11,19,24,25,30,33]+ [6,8,14,18,22,26,28,36]+[15,16,17,20,21,31,37,70]+[3,7,34,35]	0.80	0.90

5.3 MCSBGNDs-I can be constructed in Two Different Block Sizes for $v = 2ik_1 + 4k_2 - 2$ when m (mod 4) $\equiv$ 2 with

- $k_1 = 4l$, i integer & $l > 1$, (i) $k_2 = k_1 - 2$, (ii) $k_2 = 4$.
- $k_1 = 4l + 2$, (i) $k_2 = k_1 - 1$, i odd, (ii) $k_2 = k_1 - 2$, i even, (iii) $k_2 = 3$, i odd, (iv) $k_2 = 4$, i even, (v) $k_2 = 5$, i odd, $l > 1$.
- k_1 (odd) > 3:
  - (i) i (mod 4) $\equiv 0$, $k_2 = k_1 - 1$.
  - (ii) i (mod 4) $\equiv 2$, $k_2 = k_1 - 2$.
  - (iii) $k_2 = 3$, i (mod 4) $\equiv 2$.
  - (iv) $k_2 = 4$, i (mod 4) $\equiv 0$.
- k_1 (mod 4) $\equiv 1$, i (mod 4) $\equiv 3$, $k_2 = 5$.
- k_1 (mod 4) $\equiv 3$, i (mod 4) $\equiv 2$, $k_2 = 5$.

Example 8. *MCSBGNDs-I for $v = 2ik_1 + 4k_2 - 2$ with m (mod 4) $\equiv$ 2. See Table 7.*

Table 7. MCSBGNDs-I for $v = 2ik_1 + 4k_2 - 2$ with m (mod 4) $\equiv 2$.

v	k_1	k_2	Sets of Shifts	E_n	E_s
30	6	5	[2,4,5,8,11]+[3,6,7,13]+[10,12,14,15]	0.73	0.84
54	5	4	[5,9,16,20]+[8,11,13,15]+[3,12,17,21]+[19,22,26,27]+[10,18,24]+[6,23,25]	0.73	0.87
38	8	6	[2,4,9,10,14,17,19]+[11,12,13,16,18]+[3,5,7,8,15]	0.79	0.84
38	6	4	[8,10,17,18,19]+[1,3,6,12,16]+[7,14,15]	0.77	0.79
30	5	3	[5,6,7,9]+[2,4,11,13]+[14,15]+[10,12]	0.67	0.84
46	6	3	[12,14,20,21,23]+[1,5,10,11,19]+[4,6,8,9,16]+[15,18]+[17,22]	0.67	0.87
38	7	3	[2,3,5,6,8,13]+[7,9,11,14,17,18]+[15,19]+[12,16]	0.73	0.85
30	8	4	[3,6,7,8,9,12,15]+[5,10,11]+[2,13,14]	0.64	0.84
54	10	4	[2,3,8,9,10,14,18,20,24]+[11,12,13,15,16,17,19,25,27]+[6,21,26]+[5,22,23]	0.81	0.87
70	7	4	[13,16,19,23,24,33]+[10,17,25,26,27,34]+[11,18,20,22,29,35]+[3,4,8,9,14,30]+[15,21,28]+[7,31,32]	0.79	0.89
38	10	5	[3,9,10,12,13,15,16,17,19]+[5,7,8,14]+[2,6,11,18]	0.79	0.85
70	9	5	[6,10,15,16,20,21,26,28]+[5,7,12,13,22,24,25,35]+[4,14,19,23,29,30,34,63]+[8,11,18,32]+[27,31,33,36]	0.78	0.89
62	11	5	[6,9,11,13,20,22,23,25,26,29]+[4,10,12,14,18,19,21,27,30,31]+[7,8,16,28]+[5,15,17,24]	0.83	0.88

5.4 MCSBGNDs-I can be constructed in Two Different Block Sizes for $v = 2ik_1 + 4k_2 - 2$ when m (mod 4) $\equiv$ 3 with

- k_1 (mod 4) $\equiv 1$:
  - (i) i (mod 4) $\equiv 1$, $k_2 = k_1 - 1$.
  - (ii) i (mod 4) $\equiv 3$, $k_2 = k_1 - 2$.
  - (iii) i (mod 4) $\equiv 3$, $k_2 = 3$.
  - (iv) i (mod 4) $\equiv 1$, $k_2 = 4$.
  - (v) i (mod 4) $\equiv 0$, $k_2 = 5$.
- k_1 (mod 4) $\equiv 3$:
  - (i) i (mod 4) $\equiv 3$, $k_2 = k_1 - 1$.
  - (ii) i (mod 4) $\equiv 1$, $k_2 = k_1 - 2$.

(iii) $i \pmod 4 \equiv 1$, $k_2 = 3$.
(iv) $i \pmod 4 \equiv 3$, $k_2 = 4$.
(v) $i \pmod 4 \equiv 1$, $k_2 = 5$.

Example 9. *MCSBGNDs-I for $v = 2ik_1 + 4k_2 - 2$ with $m \pmod 4 \equiv 3$. See Table 8.*

Table 8. MCSBGNDs-I for $v = 2ik_1 + 4k_2 - 2$ with $m \pmod 4 \equiv 3$.

v	k_1	k_2	Sets of Shifts	E_n	E_s
48	9	8	[4,5,10,11,13,15,17,19]+[12,16,18,21,22,23,24]+[1,3,7,9, 14,20,42]	0.83	0.87
64	7	6	[10,16,18,24,25,28]+[5,13,21,27,30,32]+[9,11,12,14,20,56]+ [19,22,26,29,31]+[3,4,15,17,23]	0.81	0.88
40	5	3	[6,7,8,18]+[11,14,20,35]+[3,4,15,16]+[13,17]+[12,19]	0.70	0.86
32	7	5	[5,9,10,11,12,14]+[2,7,8,15]+[6,13,16,28]	0.74	0.84
64	9	3	[7,8,10,11,12,15]+[6,9,16,28]+[2,3,13,14]	0.76	0.88
24	7	3	[5,6,7,9,10,11]+[2,21]+[8,12]	0.67	0.82
32	9	4	[1,3,10,11,13,14,16,28]+[7,8,12]+[6,9,15]	0.67	0.84
56	7	4	[2,6,8,9,13,14,16,28]+[5,11,15]+[7,10,12]	0.77	0.88
90	9	5	[20,27,28,30,33,34,63]+[6,13,15,16,19,23,24,26]+ [5,7,8,11,12,29,32,36]+[3,21,22,25]+[10,14,17,31]	0.88	0.87
40	11	5	[3,4,9,10,11,13,15,17,18,19]+[2,8,14,16]+[7,12,20,35]	0.79	0.86

6 Generators for MCSBGNDs-I in Block of Three Different Sizes

In this Section, generators are developed to obtain MCSBGNDs-I in three different block sizes.

6.1 MCSBGNDs-I can be constructed in Three Different Block Sizes for $v = 2ik_1 + 2k_2 + 2k_3 - 2$ when $m \pmod 4 \equiv 2$ with

- $k_1 = 4l$, $k_2 = k_1 - 1$, $k_3 = k_1 - 3$, i integer.
- $k_1 = 4l + 2$:
  (i) $k_2 = k_1 - 1$, $k_3 = k_1 - 3$, i even.
  (ii) $k_2 = k_1 - 1$, $k_3 = 3$, i even.
  (iii) $k_1 = 4l + 2$, $k_2 = k_1 - 2$, $k_3 = 4$, i even.
  (iv) $k_1 = 4l + 2$, $k_2 = k_1 - 1$, $k_3 = 5$, i odd.
- k_1 (odd):
  (i) $k_2 = k_1 - 1$, $k_3 = k_1 - 3$, $i \pmod 4 \equiv 2$.
  (ii) $k_2 = k_1 - 1$, $k_3 = 3$, $i \pmod 4 \equiv 1$.
  (iii) $k_2 = k_1 - 2$, $k_3 = 4$, $i \pmod 4 \equiv 0$.
  (iv) $k_2 = k_1 - 1$, $k_3 = 5$, $i \pmod 4 \equiv 3$.
- $k_1 \pmod 4 \equiv 1$:
  (i) $i \pmod 4 \equiv 1$, $k_2 = k_1 - 1$ and $k_3 = k_1 - 2$.
  (ii) $i \pmod 4 \equiv 3$, $k_2 = k_1 - 2$, $k_3 = k_1 - 3$.
  (iii) $k_2 = k_1 - 2$, $k_3 = 3$, $i \pmod 4 \equiv 2$.
  (iv) $k_2 = k_1 - 1$, $k_3 = 4$, $i \pmod 4 \equiv 0$.
  (v) $k_2 = k_1 - 2$, $k_3 = 5$, $i \pmod 4 \equiv 0$.

- $k_1 \pmod 4 \equiv 3$:
  - (i) $i \pmod 4 \equiv 3$, $k_2 = k_1 - 1$ and $k_3 = k_1 - 2$.
  - (ii) $i \pmod 4 \equiv 1$, $k_2 = k_1 - 2$ and $k_3 = k_1 - 3$.
  - (iii) $k_2 = k_1 - 2$, $k_3 = 3$, $i \pmod 4 \equiv 0$.
  - (iv) $k_2 = k_1 - 1$, $k_3 = 4$, $i \pmod 4 \equiv 2$.
  - (v) $k_2 = k_1 - 2$, $k_3 = 5$, $i \pmod 4 \equiv 2$.

Example 10. *MCSBGNDs-I for $v = 2ik_1 + 2k_2 + 2k_3 - 2$ with $m \pmod 4 \equiv 2$. See Table 9.*

Table 9. MCSBGNDs-I for $v = 2ik_1 + 2k_2 + 2k_3 - 2$ with $m \pmod 4 \equiv 2$.

v	k_1	k_2	k_3	Sets of Shifts	E_n	E_s
22	5	4	3	[3,4,6,8]+[2,9,11]+[7,10]	0.63	0.81
62	7	6	5	[3,5,8,12,15,19]+[10,11,21,22,26,27]+[6,16,20,23,25,30]+[13,14,28,29,31]+[2,17,18,24]	0.80	0.88
38	8	7	5	[5,6,8,11,12,13,17]+[7,9,10,14,15,18]+[3,7,9,18]	0.87	0.80
70	10	9	7	[6,14,16,18,27,30,31,33,34]+[5,7,8,12,13,15,25,26,29]+[4,9,11,17,19,22,24,32]+[10,19,21,23,28,35]	0.85	0.88
62	9	8	6	[7,8,9,13,14,19,22,31]+[4,6,11,15,18,20,21,29]+[5,10,12,17,23,24,30]+[17,23,24,28,30]	0.78	0.85
78	9	7	6	[1,7,12,15,21,28,34,38]+[5,8,13,14,19,26,31,37]+[17,20,22,23,33,35,36,39]+[16,18,24,25,30,32]+[6,10,16,18,24]	0.80	0.88
30	7	5	4	[5,6,10,11,12,15]+[3,4,8,13]+[7,9,14]	0.71	0.84
38	6	5	3	[3,4,7,11,12]+[10,13,14,15,18]+[5,8,9,16]+[16,17]	0.72	0.82
62	7	6	3	[7,17,20,25,27,28]+[8,18,21,22,24,26]+[4,9,10,13,23]+[6,11,12,15,16]+[30,31]+[19,29]	0.64	0.84
54	9	7	3	[4,5,10,11,13,15,20,27]+[1,2,6,14,16,22,23,24]+[9,12,17,18,19,26]+[21,26]	0.70	0.86
70	7	5	3	[5,9,29,30,32,34]+[3,4,11,14,16,20]+[12,13,21,23,31,33]+[15,19,22,24,25,27]+[6,10,26,28]+[18,35]	0.79	0.89
94	9	8	4	[3,5,20,27,29,30,31,41]+[12,24,33,37,39,40,43,44]+[6,14,16,25,26,32,34,35]+[4,8,9,11,17,45,46,47]+[15,18,19,21,28,36,38]+[22,23, 28]	0.78	0.90
46	7	6	4	[5,9,13,17,21,23]+[10,11,14,16,18,22]+[3,6,8,12,15]+[7,19,20]	0.85	0.83
62	10	8	4	[10,14,18,20,22,23,24,26,27]+[6,8,9,11,12,13,15,19,30]+[3,5,7,21,28,29,31]+[5,28,29]	0.72	0.85
38	9	7	4	[9,11,12,14,15,16,17,18]+[1,3,4,5,6,19]+[8,10,13]	0.78	0.85
46	10	9	5	[2,3,4,6,8,10,17,20,21]+[11,14,15,16,18,19,22,23]+[7,9,11,14]	0.82	0.84
78	9	8	5	[22,23,24,25,31,33,35,37]+[16,17,19,27,32,34,38,39]+[7,10,14,18,21,26,29,30]+[2,5,8,9,11,15,28]+[5,9,28,36]	0.75	0.87
94	9	7	5	[18,27,28,31,33,39,43,46]+[1,10,21,26,29,32,34,35]+[7,8,9,16,24,36,42,44]+[6,13,14,20,22,23,41,45]+[5,25,30,38,40,47]+[5,19,30, 37]	0.85	0.89
70	11	9	5	[8,11,15,16,17,21,24,29,31,34]+[5,6,13,18,22,23,26,30,32,33]+[3,10,12,14,20,25,27,28]+[10,12,19,28]	0.90	0.81

6.2 MCSBGNDs-I can be constructed in Three Different Block Sizes for $v = 2ik_1 + 2k_2 + 2k_3 - 2$ when $m \pmod 4 \equiv 3$ with

- $k_1 = 4l$, (i) $k_2 = k_1 - 1$, $k_3 = k_1 - 2$, i integer, (ii) $k_1 = 4l$, $k_2 = k_1 - 2$, $k_3 = 3$, i integer.
- $k_1 = 4l + 2$:
  - (i) $k_2 = k_1 - 1$, $k_3 = k_1 - 2$, i even.
  - (ii) $k_2 = k_1 - 2$, $k_3 = k_1 - 3$, i odd.

(iii) $k_2 = k_1 - 2$, $k_3 = 3$, i odd.
(iv) $k_2 = k_1 - 1$, $k_3 = 4$, i even.
(v) $k_2 = k_1 - 2$, $k_3 = 5$, i even.

- k_1 (odd):
  (i) $k_2 = k_1 - 1$, $k_3 = k_1 - 2$, $i \pmod 4 \equiv 3$.
  (ii) $k_2 = k_1 - 2$, $k_3 = k_1 - 3$, $i \pmod 4 \equiv 0$.
  (iii) $k_2 = k_1 - 2$, $k_3 = 3$, $i \pmod 4 \equiv 3$.
  (iv) $k_2 = k_1 - 1$, $k_3 = 4$, $i \pmod 4 \equiv 1$.
  (v) $k_2 = k_1 - 2$, $k_3 = 5$, $i \pmod 4 \equiv 1$.
- $k_1 \pmod 4 \equiv 1$:
  (i) $i \pmod 4 \equiv 3$, $k_2 = k_1 - 1$ and $k_3 = k_1 - 3$.
  (ii) $k_2 = k_1 - 1$, $k_3 = 3$, $i \pmod 4 \equiv 2$.
  (iii) $k_2 = k_1 - 2$, $k_3 = 4$, $i \pmod 4 \equiv 2$.
  (iv) $k_2 = k_1 - 1$, $k_3 = 5$, $i \pmod 4 \equiv 0$.
- $k_1 \pmod 4 \equiv 3$:
  (i) $i \pmod 4 \equiv 1$, $k_2 = k_1 - 1$, $k_3 = k_1 - 3$.
  (ii) $k_2 = k_1 - 1$, $k_3 = 3$, $i \pmod 4 \equiv 0$.
  (iii) $k_2 = k_1 - 2$, $k_3 = 4$, $i \pmod 4 \equiv 0$.
  (iv) $k_2 = k_1 - 1$, $k_3 = 5$, $i \pmod 4 \equiv 2$.

Example 11. *MCSBGNDs-I for $v = 2ik_1 + 2k_2 + 2k_3 - 2$ with $m \pmod 4 \equiv 3$. See Table 10.*

6.3 MCSBGNDs-I can be constructed in Three Different Block Sizes for $v = 2ik_1 + 4k_2 + 2k_3 - 2$ when $m \pmod 4 \equiv 2$ with

- $k_1 = 4l$, $k_2 = k_1 - 1$, $k_3 = k_1 - 2$, i integer.
- $k_1 = 4l + 2$:
  (i) $k_2 = k_1 - 1$, $k_3 = k_1 - 2$, i odd.
  (ii) $k_2 = k_1 - 1$, $k_3 = 4$, i odd.
  (iii) $k_2 = k_1 - 2$, $k_3 = 4$, i even.
- k_1 (odd):
  (i) $k_2 = k_1 - 1$, $k_3 = k_1 - 2$, $i \pmod 4 \equiv 1$.
  (ii) $k_2 = k_1 - 1$, $k_3 = 4$, $i \pmod 4 \equiv 0$.
  (iii) $k_2 = k_1 - 2$, $k_3 = 4$, $i \pmod 4 \equiv 2$.
- $k_1 \pmod 4 \equiv 1$:
  (i) $i \pmod 4 \equiv 2$, $k_2 = k_1 - 1$, $k_3 = k_1 - 3$.
  (ii) $i \pmod 4 \equiv 0$, $k_2 = k_1 - 2$, $k_3 = k_1 - 3$.
  (iii) $k_2 = k_1 - 1$, $k_3 = 3$, $i \pmod 4 \equiv 1$.
  (iv) $k_2 = k_1 - 2$, $k_3 = 3$, $i \pmod 4 \equiv 3$.
  (v) $k_2 = k_1 - 1$, $k_3 = 5$, $i \pmod 4 \equiv 3$.
  (vi) $k_2 = k_1 - 2$, $k_3 = 5$, $i \pmod 4 \equiv 1$.
- $k_1 \pmod 4 \equiv 3$:
  (i) $i \pmod 4 \equiv 0$, $k_2 = k_1 - 1$, $k_3 = k_1 - 3$.
  (ii) $i \pmod 4 \equiv 2$, $k_2 = k_1 - 2$, $k_3 = k_1 - 3$.

Table 10. MCSBGNDs-I for $v = 2ik_1 + 2k_2 + 2k_3 - 2$ with m (mod 4) $\equiv$ 3.

v	k_1	k_2	k_3	Sets of Shifts	E_n	E_s
40	8	7	6	[4,11,14,17,18,20,35]+[7,8,12,13,15,19]+[2,6,7,12,13]	0.59	0.81
72	10	9	8	[6,14,15,21,28,30,31,33,34]+[3,11,12,13,16,17,20,22,29]+ [8,10,19,23,25,32,36,63]+[2,5,7,10,25,32,63]	0.77	0.86
64	9	8	7	[2,5,15,21,24,25,28,39]+[12,17,29,33,34,35,36,38]+ [11,13,19,20,26,27,37]+[3,9,22,23,31,32,40]+ [8,14,16,18,30,70]	0.80	0.90
80	9	8	6	[6,7,20,24,35,36,40,70]+[1,11,12,16,19,28,34,39]+ [9,13,15,17,21,23,27,30]+[8,14,18,22,25,32,37]+ [25,29,32,33,37]	0.80	0.88
32	7	6	4	[7,8,9,12,13,14]+[3,6,11,16,28]+[5,11,6]	0.74	0.80
48	10	8	7	[2,4,5,10,11,22,23,24,42]+[7,8,12,13,17,18,21]+ [8,12,15,17,20,21]	0.70	0.83
96	9	7	6	[5,6,8,17,24,42,43,45]+[13,14,15,21,25,27,32,35]+ [16,20,23,28,30,36,48,84]+[9,11,18,19,22,29,37,46]+ [4,26,33,38,44,47]+[33,34,38, 39,44]	0.85	0.89
32	5	4	3	[2,5,12,13]+[9,11,15,28]+[7,8,14]+[10,14]	0.68	0.80
72	7	6	5	[11,19,22,23,31,32]+[17,18,21,26,27,35]+[12,14,15,16,20,63]+ [7,8,28,29,33,36]+[2,5,10,24,30]+[25,34]	0.75	0.89
32	8	6	3	[3,5,7,9,10,13,16]+[2,8,12,14,28]+[12,14]	0.66	0.80
24	6	4	3	[4,5,6,11,21]+[7,8,9]+[8,9]	0.64	0.74
72	7	5	3	[11,13,15,16,23,28]+[3,9,22,25,26,27]+[2,10,14,17,19,49]+ [5,8,18,21]+[20,24]	0.72	0.88
40	6	5	4	[3,4,9,10,14]+[2,11,15,16,35]+[13,17,18,20]+[8,12, 13]	0.74	0.83
40	9	8	4	[4,6,7,8,10,11,13,18]+[1,2,12,14,15,17,19]+[9,14,15]	0.79	0.82
56	9	7	4	[2,4,8,10,12,22,25,28]+[3,5,6,13,15,17,26,27]+ [11,14,16,19,20,23]+[11,16,20]	0.71	0.84
72	7	5	4	[4,15,23,32,34,35]+[12,13,17,29,31,36]+[14,16,21,25,27,30]+ [7,8,18,22,24,63]+[5,19,20,28]+[10,26,33]	0.75	0.89
96	9	8	5	[6,10,11,16,21,38,42,47]+[13,15,26,27,28,44,46,84]+ [4,7,20,23,25,34,36,41]+[22,29,30,32,33,35,40,48]+ [9,14,18,24,31,43,45]+[3,9, 39,45]	0.75	0.90
48	7	6	5	[9,12,13,14,22,24]+[5,7,8,10,21,42]+[16,18,19,20,23]+ [1,11,17,19]	0.66	0.85
64	10	8	5	[3,5,6,10,11,14,23,26,29]+[4,9,12,13,15,21,28,32,56]+ [7,17,18,19,20,22,25]+[24,27,30,31]	0.74	0.88
40	9	7	5	[2,4,6,9,10,11,17,20]+[7,12,13,14,15,19]+[3,7,12,18]	0.65	0.82

(iii) $k_2 = k_1 - 1$, $k_3 = 3$, i (mod 4) $\equiv$ 3.
(iv) $k_2 = k_1 - 2$, $k_3 = 3$, i (mod 4) $\equiv$ 1.
(v) $k_2 = k_1 - 1$, $k_3 = 5$, i (mod 4) $\equiv$ 1.
(vi) $k_2 = k_1 - 2$, $k_3 = 5$, i (mod 4) $\equiv$ 3.

Example 12. *MCSBGNDs-I for $v = 2ik_1 + 4k_2 + 2k_3 - 2$ with m (mod 4) $\equiv$ 2. See Table 11*

6.4 MCSBGNDs-I can be constructed in Three Different Block Sizes for $v = 2ik_1 + 4k_2 + 2k_3 - 2$ when m (mod 4) $\equiv$ 3 with

- $k_1 = 4l$, (i) $k_2 = k_1 - 2$, $k_3 = k_1 - 3$, i integer, (ii) $k_1 = 4l$, $k_2 = k_1 - 1$, $k_3 = 3$, i integer.
- $k_1 = 4l + 2$:
  (i) $k_2 = k_1 - 1$, $k_3 = k_1 - 3$, i even.
  (ii) $k_2 = k_1 - 2$, $k_3 = k_1 - 3$, i odd.
  (iii) $k_2 = k_1 - 1$, $k_3 = 3$, i even.
  (iv) $k_2 = k_1 - 2$, $k_3 = 3$, i odd.

Table 11. MCSBGNDs-I for $v = 2ik_1 + 4k_2 + 2k_3 - 2$ with $m \pmod 4 \equiv 2$.

v	k_1	k_2	k_3	Sets of Shifts	E_n	E_s
54	8	7	6	[5,8,13,15,17,22,27]+[7,12,18,21,23,24]+[11,14,18,19,21,23] +[12,18,20,25,26]	0.81	0.81
38	6	5	4	[1,4,5,10,18]+[14,15,16,19]+[3,9,11,13]+[8,11,17]	0.74	0.81
30	5	4	3	[4,5,6,15]+[8,10,11]+[8,9,10]+[10,12]	0.66	0.73
78	9	8	6	[6,7,12,14,25,28,29,30]+[4,8,9,15,23,27,34,35]+ [13,16,17,19,21,26,33]+[10,11,16,21,26,33,39]+ [2,3,16,24,33]	0.79	0.85
86	7	6	4	[13,19,29,32,34,42]+[12,22,25,33,37,43]+[6,24,30,35,36,40]+ [9,17,27,31,39,41]+[5,15,16,20,26]+[7,10,18,21,28]+ [14,23,38]	0.77	0.88
110	9	7	6	[19,20,34,37,46,52,53,55]+[25,32,36,40,41,44,47,54]+ [10,15,16,23,30,31,43,49]+[6,8,13,18,39,42,45,48]+ [4,5,9,17,24,51] +[22,28,29, 38,50,51]+[12,17,22,26,33]	0.79	0.90
54	7	5	4	[6,13,17,19,23,27]+[12,14,16,18,20,21]+[5,9,15,25]+ [1,4,24,25]+[2,25,26]	0.65	0.83
30	5	4	3	[3,5,8,14]+[7,9,13]+[6,9,11]+[13,15]	0.66	0.80
70	7	6	3	[13,14,16,29,31,33]+[9,19,20,27,30,34]+[11,12,17,25,32,35]+ [6,7,10,21,23]+[2,5,15,22,26]+[24,28]	0.79	0.89
86	9	7	3	[2,8,9,10,19,40,41,42]+[7,11,12,14,20,32,33,38]+ [6,13,15,18,22,27,30,37]+[16,25,26,28,34,43]+ [21,25,28,29,34,35]+[26,43]	0.88	0.86
38	7	5	3	[6,9,11,13,16,19]+[3,5,12,18]+[1,8,14,15]+[12,18]	0.65	0.80
38	6	5	4	[11,12,13,16,17]+[4,8,9,15]+[5,6,10,15]+[5,14,15]	0.82	0.78
110	9	8	4	[2,16,18,30,31,38,42,43]+[5,8,15,19,20,45,51,53]+ [7,11,14,23,25,41,46,50]+[6,13,22,24,27,28,44,55]+ [10,21,26,29,37,39,49]+[12,17,21,26,29,52,54]+[29,32,40]	0.85	0.89
78	10	8	4	[12,17,20,22,25,30,33,37,38]+[4,5,7,9,13,24,28,29,34]+ [11,14,15,16,31,32,36]+[10,11,14,16,23,35,39]+[15,18,35]	0.84	0.87
70	9	7	4	[3,7,13,17,19,22,26,31]+[4,5,8,12,18,28,29,35]+ [10,15,20,25,30,34]+[15,16,23,25,27,34]+[10,24,27]	0.77	0.86
94	9	8	5	[8,13,14,20,27,30,37,38]+[10,11,16,17,18,21,44,47]+ [23,29,31,33,36,40,41,46]+[9,19,24,28,34,35,39]+ [9,12,15,28,34,43,45]+[6,24, 25,39]	0.77	0.88
46	7	6	5	[1,3,4,8,13,17]+[11,14,19,20,23]+[12,14,15,19,22]+ [5,7,10,22]	0.78	0.81
54	9	7	5	[8,14,18,21,23,24,25,26]+[9,12,15,16,22,27]+[2,5,6,9,15,16]+ [4,11,15,22]	0.81	0.82
110	11	9	5	[3,15,26,29,32,36,41,45,49,54]+[9,10,12,31,33,35,42,44,51,55]+ [6,13,14,22,27,46,47,48,52,53]+[17,19,23,24,25,30,37,40] +[4,5,21,30, 37,39,40,43]+[20,21,23,28]	0.81	0.89

(v) $k_2 = k_1 - 1$, $k_3 = 5$, i odd.

(vi) $k_2 = k_1 - 2$, $k_3 = 5$, i even.

- k_1 (odd):

  (i) $i \pmod 4 \equiv 3$, $k_2 = k_1 - 1$ and $k_3 = k_1 - 3$.

  (ii) $i \pmod 4 \equiv 1$, $k_2 = k_1 - 2$ and $k_3 = k_1 - 3$.

  (iii) $k_2 = k_1 - 1$, $k_3 = 3$, $i \pmod 4 \equiv 2$.

  (iv) $k_2 = k_1 - 2$, $k_3 = 3$, $i \pmod 4 \equiv 0$.

  (v) $k_2 = k_1 - 1$, $k_3 = 5$, $i \pmod 4 \equiv 0$.

  (vi) $k_2 = k_1 - 2$, $k_3 = 5$, $i \pmod 4 \equiv 2$.

- $k_1 \pmod 4 \equiv 1$:

  (i) $i \pmod 4 \equiv 2$, $k_2 = k_1 - 1$ and $k_3 = k_1 - 2$.

  (ii) $k_2 = k_1 - 1$, $k_3 = 4$, $i \pmod 4 \equiv 1$.

(iii) $k_2 = k_1 - 2$, $k_3 = 4$, $i \pmod 4 \equiv 3$.

- $k_1 \pmod 4 \equiv 3$:
  (i) $i \pmod 4 \equiv 0$, $k_2 = k_1 - 1$ and $k_3 = k_1 - 2$.
  (ii) $k_2 = k_1 - 1$, $k_3 = 4$, $i \pmod 4 \equiv 3$.
  (iii) $k_2 = k_1 - 2$, $k_3 = 4$, $i \pmod 4 \equiv 1$.

Example 13. *MCSBGNDs-I for $v = 2ik_1 + 4k_2 + 2k_3 - 2$ with $m \pmod 4 \equiv 3$. See Table 12.*

Table 12. MCSBGNDs-I for $v = 2ik_1 + 4k_2 + 2k_3 - 2$ with $m \pmod 4 \equiv 3$.

v	k_1	k_2	k_3	Sets of Shifts	E_n	E_s
40	5	4	3	[6,8,9,15]+[4,7,12,14]+[16,18,35]+[1,19,20]+[13,17]	0.69	0.86
88	7	6	5	[15,21,28,29,35,39]+[13,17,18,34,43,44]+ [19,22,25,26,37,42]+[4,20,32,38,40,41]+ [3,6,23,24,30]+[12,23,27,31,77]+[24,31,36,77]	0.81	0.88
88	10	9	7	[4,6,10,13,16,20,25,39,41]+[5,8,19,27,28,29,32,36,77]+ [18,23,26,30,31,33,42,44]+[7,9,12,21,22,23,38,44]+ [14,17,30,37,38,40]	0.82	0.88
96	9	8	6	[3,5,9,25,26,37,40,46]+[15,16,18,19,33,42,47,84]+ [6,13,20,22,28,31,34,36]+[29,35,39,41,44,45,48]+ [4,8,23,32,39,41,45]+[27,30,32, 39,43]	0.85	0.88
48	8	6	5	[5,10,11,14,15,18,20]+[2,8,19,24,42]+[9,16,21,22,24]+ [4,9,16,17]	0.86	0.82
64	10	8	7	[2,4,7,9,12,14,21,27,32]+[5,10,16,17,22,26,29]+ [6,13,18,19,20,22,29]+[13,16,19,20,28,29]	0.83	0.83
56	9	7	6	[12,16,18,20,21,22,23,26]+[6,8,14,15,17,49]+ [1,11,13,14,24,49]+[4,9,11,14,15]	0.76	0.82
48	8	7	3	[3,9,12,15,16,17,22]+[1,4,8,18,23,42]+[4,7,19,20,21,24]+ [18,23]	0.72	0.82
48	6	5	3	[12,15,18,20,24]+[3,4,5,16,19]+[9,21,22,42]+[14,23]+ [14,23]	0.80	0.82
56	7	6	3	[10,12,18,19,22,27]+[6,9,11,13,23,49]+[15,17,21,26,28]+ [3,5,14,15,17]+[25,26]	0.78	0.83
32	6	4	3	[1,5,14,16,28]+[6,11,13]+[7,10,13]+[11,13]	0.72	0.73
80	7	5	3	[11,12,26,30,33,40]+[17,19,24,28,31,35]+[7,13,14,16,36,70]+ [7,13,14,16,36,70]+[9,15,20,34]+[15,18,20,27]+[21,39]	0.79	0.85
56	9	8	4	[2,8,11,13,15,18,21,23]+[4,5,14,16,22,25,26]+ [10,12,19,20,27,28,49]+[3,26,27]	0.74	0.79
72	7	6	4	[1,4,5,11,16,35]+[10,13,22,26,34,36]+[14,17,25,27,28,31]+ [7,15,23,30,63]+[8,15,23,29,63]+[15,21,29]	0.80	0.85
88	9	7	4	[22,23,28,29,31,32,35,43]+[2,4,8,9,18,19,39,77]+ [7,10,13,15,17,34,37,38]+[6,24,27,36,40,42]+ [14,24,26,30,36,40]+[25,27,33]	0.76	0.88
40	7	5	4	[10,11,12,13,14,20]+[6,7,8,18]+[6,16,19,35]+[4,15,18]	0.67	0.83
64	10	9	5	[11,12,15,17,21,24,26,27,29]+[1,6,13,14,16,19,28,31]+ [7,13,14,16,20,30,32,56]+[6,14,20,23]	0.84	0.83
112	9	8	5	[22,27,36,42,45,48,51,53]+[19,20,26,30,38,43,52,98]+ [3,9,21,31,32,33,41,54]+[7,16,24,25,29,35,37,49]+ [8,11,13,40,44,47,56]+[5,17,18,28,46,50,56]+[4,13,44,50]	0.85	0.89
80	10	8	5	[5,7,9,12,14,22,27,28,35]+[8,13,15,16,24,25,26,39,70]+ [6,11,18,21,31,34,37]+[6,11,18,20,31,33,38]+[33,34,37,38]	0.76	0.85
72	9	7	5	[2,8,12,14,17,24,32,34]+[4,13,15,18,21,22,23,28]+ [10,19,20,25,29,35]+[10,16,19,27,29,36]+[6,10,20,33]	0.78	0.85

6.5 MCSBGNDs-I can be constructed in Three Different Block Sizes for $v = 2ik_1 + 2k_2 + 4k_3 - 2$ when $m \pmod 4 \equiv 2$ with

- $k_1 = 4l$, (i) $k_2 = k_1 - 2$, $k_3 = k_1 - 3$, i integer, (ii) $k_2 = k_1 - 2$, $k_3 = 3$, i integer.
- $k_1 = 4l + 2$:
  - (i) $k_2 = k_1 - 2$, $k_3 = k_1 - 3$, i odd.
  - (ii) $k_2 = k_1 - 2$, $k_3 = 3$, i odd.
  - (iii) $k_2 = k_1 - 2$, $k_3 = 4$, i even.
  - (iv) $k_2 = k_1 - 1$, $k_3 = 5$, i odd.
- k_1 (odd):
  - (i) $i \pmod 4 \equiv 1$, $k_2 = k_1 - 2$ and $k_3 = k_1 - 3$,
  - (ii) $k_2 = k_1 - 2$, $k_3 = 3$, $i \pmod 4 \equiv 3$.
  - (iii) $k_2 = k_1 - 2$, $k_3 = 4$, $i \pmod 4 \equiv 1$.
  - (iv) $k_2 = k_1 - 2$, $k_3 = 5$, $i \pmod 4 \equiv 3$.
- $k_1 \pmod 4 \equiv 1$:
  - (i) $k_2 = k_1 - 1$, $k_3 = k_1 - 2$ and $i \pmod 4 \equiv 2$.
  - (ii) $k_2 = k_1 - 1$, $k_3 = k_1 - 3$ and $i \pmod 4 \equiv 0$.
  - (iii) $k_2 = k_1 - 1$, $k_3 = 3$, $i \pmod 4 \equiv 2$.
  - (iv) $k_2 = k_1 - 1$, $k_3 = 4$, $i \pmod 4 \equiv 0$.
  - (v) $k_2 = k_1 - 1$, $k_3 = 5$, $i \pmod 4 \equiv 2$.
- $k_1 \pmod 4 \equiv 3$:
  - (i) $k_2 = k_1 - 1$, $k_3 = k_1 - 2$ and $i \pmod 4 \equiv 0$.
  - (ii) $k_2 = k_1 - 1$, $k_3 = k_1 - 3$ and $i \pmod 4 \equiv 2$.
  - (iii) $k_2 = k_1 - 1$, $k_3 = 3$, $i \pmod 4 \equiv 0$.
  - (iv) $k_2 = k_1 - 1$, $k_3 = 4$, $i \pmod 4 \equiv 2$.
  - (v) $k_2 = k_1 - 1$, $k_3 = 5$, $i \pmod 4 \equiv 0$.

Example 14. *MCSBGNDs-I for $v = 2ik_1 + 2k_2 + 4k_3 - 2$ with $m \pmod 4 \equiv 2$. See Table 13.*

6.6 MCSBGNDs-I can be constructed in Three Different Block Sizes for $v = 2ik_1 + 2k_2 + 4k_3 - 2$ when $m \pmod 4 \equiv 3$ with

- $k_1 = 4l$, (i) $k_2 = k_1 - 1$, $k_3 = k_1 - 3$, i integer, (ii) $k_2 = k_1 - 1$, $k_3 = 3$, i integer.
- $k_1 = 4l + 2$:
  - (i) $k_2 = k_1 - 1$, $k_3 = k_1 - 2$, i even.
  - (ii) $k_2 = k_1 - 1$, $k_3 = k_1 - 3$, i odd.
  - (iii) $k_2 = k_1 - 1$, $k_3 = 3$, i odd.
  - (iv) $k_2 = k_1 - 1$, $k_3 = 4$, i even.
  - (v) $k_2 = k_1 - 1$, $k_3 = 5$, i odd.
- k_1 (odd):
  - (i) $i \pmod 4 \equiv 3$, $k_2 = k_1 - 1$ and $k_3 = k_1 - 2$.
  - (ii) $i \pmod 4 \equiv 1$, $k_2 = k_1 - 1$ and $k_3 = k_1 - 3$.
  - (iii) $k_2 = k_1 - 1$, $k_3 = 3$, $i \pmod 4 \equiv 3$.
  - (iv) $k_2 = k_1 - 1$, $k_3 = 4$, $i \pmod 4 \equiv 1$.

Table 13. MCSBGNDs-I for $v = 2ik_1 + 2k_2 + 4k_3 - 2$ with $m \pmod 4 \equiv 2$.

v	k_1	k_2	k_3	Sets of Shifts	E_n	E_s
78	9	8	7	[10,12,15,16,19,20,29,33]+[7,8,13,14,18,26,30,39]+ [6,9,11,25,28,36,38]+[5,11,31,34,37,38]+[5,17,25,34,35,37]	0.84	0.86
86	7	6	5	[15,21,22,24,37,39]+[10,27,30,31,34,35]+[1,4,9,11,28,33]+ [16,18,20,32,41,42]+[23,25,29,40,43]+[26,38,40,43]+[7,8,26,43]	0.77	0.88
110	9	8	6	[10,14,17,19,21,32,49,51]+[16,18,22,24,25,26,40,43]+ [5,37,41,42,46,48,52,55]+[27,29,35,36,39,47,50,54]+ [15,23,31,33,34,38,44]+[3,15,23,31,38]+[2,8,11,44,45]	0.79	0.89
94	11	10	8	[11,14,22,23,24,26,31,35,44,47]+ [9,15,19,21,27,30,36,40,41,42]+[3,8,10,13,18,25,32,33,46]+ [4,7,28,33,34,39,43]+[4,16,17,29,37,39,46]	0.73	0.88
46	8	6	5	[6,11,15,16,22,26,27]+[12,13,14,17,20,23,25]+ [8,24,29,30,31]+[5,7,18,28]+[4,5,21,30]	0.81	0.86
30	6	4	3	[2,4,5,6,12]+[8,9,13]+[10,11]+[8,15]	0.66	0.80
54	9	7	6	[3,5,10,12,13,14,25,26]+[8,15,18,19,22,24]+ [16,17,18,19,23]+[4,6,9,15,19]	0.70	0.82
38	5	4	3	[5,8,9,12]+[2,3,13,19]+[7,10,15]+[11,17]+[11,17]	0.76	0.79
78	7	6	3	[15,18,20,24,34,39]+[11,23,27,29,30,32]+ [14,17,25,26,28,37]+[1,3,10,12,19,33]+[16,22,31,36,38]+ [22,35]+[35,38]	0.75	0.87
38	8	6	3	[3,6,9,12,13,14,18]+[8,11,16,17,19]+[15,19]+[15,19]	0.81	0.79
46	10	8	3	[3,4,5,8,9,13,14,15,21]+[6,7,10,11,16,17,23]+[16,23]+[16,23]	0.77	0.78
78	9	7	3	[6,9,16,21,22,24,26,28]+[2,7,13,19,23,25,32,34]+ [11,12,14,15,18,20,30,33]+[8,17,29,31,35,36]+[36,37]+[36,37]	0.77	0.87
102	9	8	4	[7,9,20,23,26,29,44,45]+[4,10,14,16,35,37,41,47]+ [3,12,17,25,27,33,42,43]+[6,8,13,21,30,34,36,51]+ [24,28,39,46,48,49,50]+[19,22, 50]+[19,32,40]	0.74	0.89
54	7	6	4	[8,10,12,23,25,27]+[9,13,16,21,22,26]+[2,4,11,17,20]+ [11,17,20]+[11,19,20]	0.77	0.81
70	10	8	4	[3,4,6,9,10,22,23,29,34]+[2,7,11,12,13,14,24,26,30]+ [8,15,16,18,19,27,32]+[16,21,25]+[16,19,27]	0.78	0.85
46	9	7	4	[2,4,6,8,9,19,20,23]+[5,11,15,18,21,22]+[12,14,17]+[12,13,14]	0.71	0.84
70	9	8	5	[4,7,9,13,21,22,27,35]+[14,16,18,28,30,31,32,33]+ [10,11,17,23,24,26,29]+[10,11,20,26]+[5,15,24,26]	0.73	0.85
86	7	6	5	[12,22,26,28,32,41]+[2,29,30,34,36,40]+[15,16,27,33,35,42]+ [13,24,25,31,37,39]+[9,10,17,20,23]+[6,14,23,43]+[8,14,20,38]	0.77	0.88
54	10	8	5	[1,2,6,7,12,16,18,20,26]+[5,9,13,14,19,21,23]+[19,23,24,25]+ [5,10,17,19]	0.70	0.83
86	9	7	5	[9,11,14,18,23,24,27,38]+[4,5,12,15,19,31,40,43]+ [1,7,17,20,26,28,36,37]+[16,21,22,34,35,42]+[13,16,21,34]+ [13,16,21,34]	0.79	0.85

(v) $k_2 = k_1 - 1$, $k_3 = 5$, $i \pmod 4 \equiv 3$.

- $k_1 \pmod 4 \equiv 1$:
  - (i) $k_2 = k_1 - 2$, $k_3 = k_1 - 3$ and $i \pmod 4 \equiv 2$.
  - (ii) $k_2 = k_1 - 2$, $k_3 = 3$, $i \pmod 4 \equiv 0$.
  - (iii) $k_2 = k_1 - 2$, $k_3 = 4$, $i \pmod 4 \equiv 2$.
  - (iv) $k_2 = k_1 - 2$, $k_3 = 5$, $i \pmod 4 \equiv 0$.
- $k_1 \pmod 4 \equiv 3$:
  - (i) $k_2 = k_1 - 2$, $k_3 = k_1 - 3$ and $i \pmod 4 \equiv 0$.
  - (ii) $k_2 = k_1 - 2$, $k_3 = 3$, $i \pmod 4 \equiv 2$.
  - (iii) $k_2 = k_1 - 2$, $k_3 = 4$, $i \pmod 4 \equiv 0$.
  - (iv) $k_2 = k_1 - 2$, $k_3 = 5$, $i \pmod 4 \equiv 2$.

Example 15. *MCSBGNDs-I for $v = 2ik_1 + 2k_2 + 4k_3 - 2$ with $m \pmod 4 \equiv 3$. See Table 14.*

Table 14. MCSBGNDs-I for $v = 2ik_1 + 2k_2 + 4k_3 - 2$ with $m \pmod 4 \equiv 3$.

v	k_1	k_2	k_3	Sets of Shifts	E_n	E_s
48	6	5	4	[1,16,17,20,42]+[3,4,7,8,24]+[19,21,22,23]+[9,12,22]+ [11,13,14]	0.74	0.85
48	5	4	3	[8,9,13,14]+[10,19,22,42]+[1,12,17,18]+[7,15,21]+[16,21]+ [16,21]	0.70	0.82
48	8	7	5	[7,8,10,12,14,20,21]+[9,11,13,15,23,24]+[3,9,11,24]+ [11,16,22,42]	0.78	0.81
32	6	5	3	[8,10,11,13,15]+[5,14,16,28]+[14,16]+[3,28]	0.79	0.77
56	9	8	6	[11,13,14,16,19,22,23,49]+[5,6,15,17,18,21,28]+ [4,8,9,17,18]+[12,18,24,27,28]	0.70	0.84
72	9	7	6	[10,14,17,20,26,28,35,63]+[2,7,8,16,21,24,31,34]+ [5,11,23,32,33,36]+[4,5,13,18,32]+[18,19,27,29,36]	0.82	0.86
80	7	5	4	[12,16,27,29,32,35]+[18,20,22,25,33,40]+[4,5,11,13,14,30]+ [6,7,15,23,38,70]+[8,17,21,34]+[17,21,34]+[19,24,37]	0.71	0.87
40	8	7	3	[4,7,8,12,14,15,18]+[3,6,9,11,16,35]+[11,20]+[11, 20]	0.72	0.77
48	10	9	3	[4,5,8,11,13,14,23,24,42]+[2,3,9,10,12,18,19,22]+[15,21]+ [18,20]	0.79	0.84
64	7	6	3	[4,9,10,16,31,56]+[12,15,21,22,25,32]+[6,19,20,23,26,29]+ [17,18,24,27,28]+[24,27]+[27,20]	0.83	0.85
96	9	7	3	[7,16,18,20,21,25,37,45]+[4,6,13,24,33,34,35,41]+ [8,10,19,27,29,30,32,36+[5,11,23,39,40,42,44,84]+ [14,22,26,28,46,47]+[28,46]+[28,46]	0.83	0.87
48	7	5	3	[9,15,16,17,19,20]+[4,8,10,13,18,42]+[5,7,11,22]+[22,23]+ [22,23]	0.68	0.80
72	10	9	4	[10,12,17,19,21,29,31,33,36]+[2,7,16,18,22,25,28,34,63]+ [6,11,13,14,20,24,26,27]+[14,15,30]+[14,15,30]	0.82	0.85
48	9	8	4	[5,7,8,10,13,16,17,19]+[12,14,15,18,20,23,42]+[11,15,22]+ [12,15,18]	0.71	0.81
64	9	7	4	[6,9,12,15,16,17,25,27]+[10,11,18,19,24,26,28,56]+ [5,13,14,29,31,32]+[13,20,29]+[13,20,29]	0.81	0.83
120	11	9	4	[9,11,28,32,39,40]+[12,21,26,27,30,38]+[17,20,22,31,33,35]+ [15,18,25,29,34,36]+[13,14,16,37]+[19,24,37]+[14,24,37]	0.71	0.87
56	10	9	5	[3,5,8,11,13,15,16,18,21]+[1,9,17,20,23,24,25,49]+ [6,10,14,25]+[6,9,14,23]	0.81	0.83
88	9	7	5	[5,10,22,23,24,26,28,34]+[7,12,16,18,19,31,33,38]+ [13,17,20,21,29,40,44,77]+[9,14,15,25,27,35,43]+ [32,36,37,41]+[1,6,39,42]	0.80	0.90
104	9	7	5	[5,7,12,14,34,36,44,52]+[9,16,17,21,28,31,32,51]+ [10,15,25,30,42,45,48,91]+[23,27,29,37,38,40,49,50]+ [18,26,35,39,43,47]+[11,24,33,35]+[11,24,26,41]	0.81	0.90
80	7	6	5	[3,15,16,18,19,23,32,35,38,40]+ [9,11,14,17,20,24,25,36,37,39]+ [4,7,12,21,26,28,29,31]+[22,30,33,70]+[6,13,27,34]	0.84	0.90

6.7 MCSBGNDs-I can be constructed in Three Different Block Sizes for $v = 2ik_1 + 4k_2 + 4k_3 - 2$ when $m \pmod 4 \equiv 2$ with

- $k_1 = 4l$:

  (i) $k_2 = k_1 - 1$, $k_3 = k_1 - 3$, i integer.

  (ii) $k_2 = k_1 - 1$, $k_3 = 3$, i integer.

  (iii) $k_2 = k_1 - 1$, $k_3 = 4$, i integer.

- $k_1 = 4l + 2$:
  - (i) $k_2 = k_1 - 1$, $k_3 = k_1 - 2$, i odd.
  - (ii) $k_2 = k_1 - 1$, $k_3 = k_1 - 3$, i even.
  - (iii) $k_2 = k_1 - 2$, $k_3 = k_1 - 3$, i odd.
  - (iv) $k_2 = k_1 - 1$, $k_3 = 3$, i even.
  - (v) $k_2 = k_1 - 2$, $k_3 = 3$, i odd.
  - (vi) $k_2 = k_1 - 1$, $k_3 = 4$, i odd.
  - (vii) $k_2 = k_1 - 2$, $k_3 = 4$, i even.
  - (viii) $k_2 = k_1 - 1$, $k_3 = 5$, i even.
  - (ix) $k_2 = k_1 - 2$, $k_3 = 5$, i even.
- k_1 (odd):
  - (i) $i \pmod 4 \equiv 2$, $k_2 = k_1 - 1$ and $k_3 = k_1 - 2$.
  - (ii) $i \pmod 4 \equiv 0$, $k_2 = k_1 - 1$ and $k_3 = k_1 - 3$.
  - (iii) $i \pmod 4 \equiv 2$, $k_2 = k_1 - 2$ and $k_3 = k_1 - 3$.
  - (iv) $k_2 = k_1 - 1$, $k_3 = 3$, $i \pmod 4 \equiv 2$.
  - (v) $k_2 = k_1 - 2$, $k_3 = 3$, $i \pmod 4 \equiv 0$.
  - (vi) $k_2 = k_1 - 1$, $k_3 = 4$, $i \pmod 4 \equiv 0$.
  - (vii) $k_2 = k_1 - 2$, $k_3 = 4$, $i \pmod 4 \equiv 2$.
  - (viii) $k_2 = k_1 - 2$, $k_3 = 5$, $i \pmod 4 \equiv 2$.
  - (ix) $k_2 = k_1 - 2$, $k_3 = 5$, $i \pmod 4 \equiv 0$.

Example 16. *MCSBGNDs-I for $v = 2ik_1 + 4k_2 + 4k_3 - 2$ with $m \pmod 4 \equiv 2$. See Table 15.*

6.8 MCSBGNDs-I can be constructed in Three Different Block Sizes for $v = 2ik_1 + 4k_2 + 4k_3 - 2$ when $m \pmod 4 \equiv 3$ with

- $k_1 \pmod 4 \equiv 1$:
  - (i) $k_2 = k_1 - 1$, $k_3 = k_1 - 2$ and $i \pmod 4 \equiv 3$.
  - (ii) $k_2 = k_1 - 1$, $k_3 = k_1 - 3$ and $i \pmod 4 \equiv 1$.
  - (iii) $k_2 = k_1 - 2$, $k_3 = k_1 - 3$ and $i \pmod 4 \equiv 3$.
  - (iv) $k_2 = k_1 - 2$, $k_3 = k_1 - 3$ and $i \pmod 4 \equiv 1$.
  - (v) $k_2 = k_1 - 1$, $k_3 = 3$ and $i \pmod 4 \equiv 3$.
  - (vi) $k_2 = k_1 - 2$, $k_3 = 3$ and $i \pmod 4 \equiv 1$.
  - (vii) $k_2 = k_1 - 1$, $k_3 = 4$ and $i \pmod 4 \equiv 1$.
  - (viii) $k_2 = k_1 - 2$, $k_3 = 4$ and $i \pmod 4 \equiv 3$.
  - (ix) $k_2 = k_1 - 1$, $k_3 = 5$ and $i \pmod 4 \equiv 3$.
  - (x) $k_2 = k_1 - 2$, $k_3 = 5$ and $i \pmod 4 \equiv 1$.
- $k_1 \pmod 4 \equiv 3$:
  - (i) $k_2 = k_1 - 1$, $k_3 = k_1 - 2$ and $i \pmod 4 \equiv 1$.
  - (ii) $k_2 = k_1 - 1$, $k_3 = k_1 - 3$ and $i \pmod 4 \equiv 3$.
  - (iii) $k_2 = k_1 - 1$, $k_3 = 3$ and $i \pmod 4 \equiv 1$.
  - (iv) $k_2 = k_1 - 2$, $k_3 = 3$ and $i \pmod 4 \equiv 3$.
  - (v) $k_2 = k_1 - 1$, $k_3 = 4$ and $i \pmod 4 \equiv 3$.

(vi) $k_2 = k_1 - 2$, $k_3 = 4$ and $i \pmod 4 \equiv 1$.
(vii) $k_2 = k_1 - 1$, $k_3 = 5$ and $i \pmod 4 \equiv 1$.
(viii) $k_2 = k_1 - 2$, $k_3 = 5$ and $i \pmod 4 \equiv 3$.

Table 15. MCSBGNDs-I for $v = 2ik_1 + 4k_2 + 4k_3 - 2$ with $m \pmod 4 \equiv 2$.

v	k_1	k_2	k_3	Sets of Shifts	E_n	E_s
40	8	7	6	[12,14,19,21,23]+[8,9,11,16]+[5,10,13,17]+[6,18,22]+[7,15,20]	0.73	0.87
46	5	4	3	[4,9,10,21]+[5,12,14,15]+[6,17,20]+[7,18,20]+[16,22]+[16,23]	0.69	0.83
62	8	7	5	[8,11,13,16,19,24,26]+[6,10,17,28,29,30]+[9,15,21,23,27,29]+[6,10,21,23]+[4,10,21,27]	0.74	0.81
54	6	5	3	[16,20,21,23,25]+[11,17,22,24,26]+[5,14,15,18]+[6,7,10,27]+[15,27]+[15,27]	0.80	0.82
94	7	6	4	[22,23,30,34,36,43]+[18,21,27,29,32,45]+[9,13,31,38,44,46]+[15,17,28,33,40,41]+[4,10,12,25,42]+[3,10,11,26,42]+[11,39,42]+[10,37,39]	0.81	0.87
62	10	8	3	[2,7,8,9,14,15,21,22,25]+[6,10,11,19,23,26,29]+[5,6,12, 18,23,27,29]+[29,30]+[29,30]	0.80	0.82
86	9	7	6	[3,5,8,22,32,33,34,35]+[7,12,16,17,18,28,31,39]+[13,24,29,30,37,38]+[10,19,25,29,37,43]+[11,29,38,42,43]+[26,29,30,36,40]	0.83	0.85
54	8	7	3	[3,8,11,18,21,22,25]+[6,10,17,20,23,27]+[12,15,16,19,20,24]+[20,24]+[20,24]	0.77	0.79
86	10	9	3	[10,12,24,29,32,33,34,41,43]+[8,9,11,14,18,20,23,26,38]+[19,21,27,28,36,39,40,42]+[2,3,6,21,28,30,39,42]+[31,36]+[31,39]	0.79	0.86
62	7	6	3	[10,12,24,29,32,33,34,41,43]+[8,9,11,14,18,20,23,26,38]+[19,21,27,28,36,39,40,42]+[2,3,6,21,28,30,39,42]+[31,36]+[31,39]	0.76	0.79
38	6	4	3	[10,11,13,15,19]+[7,9,17]+[7,12,14]+[14,17]+[14,17]	0.71	0.75
86	7	5	3	[18,20,23,33,34,43]+[5,6,10,12,17,32]+[9,19,31,35,36,39]+[13,24,27,28,38,42]+[7,11,29,37]+[11,14,22,37]+[25,40]+[25,40]	0.79	0.87
54	8	6	4	[5,8,12,13,20,24,26]+[15,17,19,21,25]+[2,7,9,14,21]+[10,18,23]+[14,16,17]	0.77	0.83
46	6	5	4	[4,6,10,12,14]+[7,8,11,18]+[7,8,13,15]+[8,13,18]+[8,13,18]	0.73	0.72
118	9	8	4	[24,30,38,46,48,52,53,58]+[8,14,21,22,29,39,45,56]+[16,23,25,27,28,34,35,47]+[7,10,13,26,36,42,44,55]+[9,12,32,33,40,50,54]+[15,17,19,32,33,40,50,54]+[11,41,57]+[19,41,49]	0.88	0.89
86	10	8	4	[16,17,19,22,27,32,33,41,43]+[14,21,24,25,29,30,36,39,40]+[2,3,4,6,13,20,37]+[10,11,20,23,26,37,38]+[10,31,38]+[9,35,37]	0.79	0.87
78	9	7	4	[13,15,19,31,34,35,37,39]+[3,6,7,17,27,30,32,33]+[14,20,22,26,28,36]+[4,5,9,14,18,26]+[18,23,25]+[20,23,25]	0.77	0.86
94	10	9	5	[8,9,28,30,33,40,41,44,45]+[1,6,7,18,22,26,27,35,46]+[5,11,13,20,21,32,37,47]+[13,25,31,36,37,38,43,47]+[12,16,19,42]+[12,15,21,36]	0.81	0.87
86	9	8	5	[4,13,14,16,20,31,33,41]+[21,23,25,27,34,37,38,43]+[11,15,18,24,28,32,42]+[8,9,19,24,29,35,42]+[9,17,24,35]+[7,9,24,40]	0.80	0.85
70	10	8	5	[13,14,18,20,33,39,43,45,46]+[3,6,9,21,26,27,28,31,37]+[4,5,7,19,22,25,30,36,38]+[15,16,17,24,29,34,41]+[32,42,44,47]+[8,10,35,40]	0.85	0.90
118	9	7	5	[4,14,23,34,36,39,42,43]+[10,18,25,27,32,33,38,50]+[7,13,16,30,35,44,45,46]+[20,21,28,47,51,55,56,57]+[9,11,40,53,58,59]+[12,22,41,48,53,54]+[8,15,41,48]+[6,22,31,54]	0.85	0.89

Example 17. *MCSBGNDs-I for $v = 2ik_1 + 4k_2 + 4k_3 - 2$ with m (mod 4) $\equiv 3$. See Table 16.*

Table 16. MCSBGNDs-I for $v = 2ik_1 + 4k_2 + 4k_3 - 2$ with m (mod 4) $\equiv 3$.

v	k_1	k_2	k_3	Sets of Shifts	E_n	E_s
56	5	4	3	[5,12,14,21]+[3,16,17,18]+[20,22,23,28]+[10,13,25]+ [9,15,26]+[6,49]+[6,49]	0.77	0.84
56	7	6	5	[8,15,16,19,24,25]+[13,20,22,26,28]+[11,21,23,26,28]+ [10,11,12,23]+[6,9,11,26]	0.77	0.81
72	9	8	6	[8,19,20,21,25,27,33,63]+[10,12,14,17,24,28,32]+ [5,11,15,18,26,32,34]+[18,24,28,29,34]+[13,29,32,34,35]	0.83	0.84
80	7	6	4	[8,22,23,34,35,37]+[5,9,12,13,17,24]+[6,7,11,28,36,70]+ [25,26,27,39,40]+[19,26,27,31,39]+[19,27,31]+[18,27,32]	0.79	0.84
104	9	7	6	[19,24,25,28,33,40,49,91]+[4,8,16,31,32,37,38,42]+ [20,26,29,34,44,46,47,48]+[6,7,10,12,22,45]+ [12,23,35,36,41,50]+[6,7,22,23, 41]+[23,35,36,41,52]	0.87	0.84
48	7	5	6	[9,10,14,18,19,24]+[1,11,16,20]+[13,17,23,42]+[11,13,16]+ [11,12,17]	0.74	0.80
96	9	8	3	[3,6,8,18,31,32,46,47]+[11,13,19,22,24,26,37,38]+ [5,7,14,16,17,40,44,45]+[20,25,30,36,41,42,84]+ [9,21,23,29,30,39,41]+[25,48]+[35,41]	0.83	0.88
48	7	6	3	[2,3,4,7,9,22]+[14,15,18,19,20]+[5,12,13,24,42]+[16,21]+ [17,23]	0.75	0.87
56	9	7	3	[3,5,9,10,16,17,25,26]+[14,19,22,23,28,49]+ [11,14,15,19,23,24]+[19,22]+[15,27]	0.84	0.80
72	7	5	3	[17,24,31,33,34,63]+[18,19,20,23,26,28]+[3,4,6,8,22,29]+ [25,30,32,36]+[7,13,21,30]+[32,35]+[21,36]	0.73	0.86
64	9	8	4	[4,19,25,26,28,29,30,31]+[9,10,14,18,20,22,32]+ [15,17,20,22,24,32,56]+[17,18,22]+[15,17,18]	0.81	0.81
120	11	10	4	[3,8,9,16,17,22,25,39,42,57]+[6,18,19,35,37,38,41,51,55,56]+ [10,12,20,27,40,45,46,47,54,58]+[5,13,21,28,30,31,32,36,44]+ [21,28,29,33,43,44,49,50,52]+[23,43,49]+[32,33,44]	0.83	0.89
96	9	7	4	[9,11,16,25,43,46,47,84]+[2,10,13,23,30,32,6,45]+ [6,14,15,20,27,28,37,42]+[17,22,33,35,41,44]+ [17,22,33,35,41,44]+[22,34,40]+[22,26,40]	0.75	0.86
48	7	5	4	[10,11,13,16,17,21]+[3,7,14,22]+[12,14,19,42]+[5,15,24]+ [12,14,19]	0.81	0.81
104	9	8	5	[9,10,20,21,24,32,39,45]+[17,18,25,26,35,40,46,91]+ [15,31,33,36,47,48,49,50]+[5,19,27,28,34,43,52]+ [11,12,22,29,38,43,52]+[4,27, 34,38]+[11,27,30,34]	0.81	0.87
80	11	10	5	[9,11,13,17,24,27,28,32,35,40]+[1,3,6,8,14,25,31,33,39]+ [7,20,21,22,23,30,36,37,39]+[16,29,31,70]+[22,30,33,70]	0.85	0.86
64	9	7	5	[6,9,11,14,17,19,20,29]+[12,13,15,26,27,31]+ [15,25,27,30,32,56]+[10,26,31,56]+[15,24,28,56]	0.86	0.82
120	11	9	5	[8,12,16,34,35,38,39,54,59,60]+ [11,18,21,36,40,42,43,45,46,52]+ [4,17,24,30,31,44,48,49,55,57]+[9,13,23,25,26,33,53,58]+ [2,3,7,20,22, 23,58,105]+[23,27,28,33]+[10,29,33,41]	0.81	0.89

7 Conclusion

In this article, some new generators have been developed to generate sets of cyclic shifts for MCSBGNDs-I when m (mod 4) $\equiv 2$ & 3 with v even for $k = 4l$, $k = 4l + 2$, k (odd) > 3, k (mod) $\equiv 1$ and k (mod) $\equiv 3$. Efficiency of neighbor effects and of Separability show that our proposed generators produce designs which control the neighbor effects efficiently as well as estimate the direct effects and neighbor effects independently.

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Abdul Salam: Conceptualization. **Zubia Batool Shamshi**: Methodology, Investigation, Writing- Reviewing. **Sajid Hussain**: Data curation, Writing- Original draft preparation, Software, Supervision. **Hurria Ali**: Visualization, Investigation. **Jamshaid ul Hassan**: Software, Validation. **Abid Khan**: Writing- Reviewing and Editing.

Compliance with Ethical Standards

It is declare that all authors don't have any conflict of interest. It is also declare that this article does not contain any studies with human participants or animals performed by any of the authors. Furthermore, informed consent was obtained from all individual participants included in the study.

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